

Experimental Conceptualisation of the Flow Net System Construction inside the Body of Homogeneous Earth Embankment Dams

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ABSTRACT

By means of a drainage and seepage tank, an experimental flow net system inside the body of a homogeneous earth embankment dam model, formed from Leighton Buzzard Silica sand, was developed and studied in this experimental research paper.

Water flow through dams is one of the basic problems for geotechnical engineers. Seepage analysis is an important factor to be considered in the proper design of many civil engineering structures. Seepage can occur in both through the structure itself as the case of earth dams and under foundations of an engineering structure. Successful seepage analysis is achieved on the proper and accurate construction of a flow net.

Amongst the various existing methods of seepage analysis, the “Finite Element Method” and the method of “Experimental Flow Nets” are the most widely used ones.

Construction of a flow net is mainly used for solving water flow problems through porous media where the geometry makes sometimes analytical solutions impractical. This method is usually used in soil mechanics, geotechnical or civil engineering as an initial check for problems of water flow under hydraulic structures like embankments or dams. As such, a grid obtained by drawing a series of equipotential lines and stream or flow lines is called a flow net. In this procedure the Laplace equation principles must be satisfied.

Hence, the construction of a flow net is an important tool in analysing two-dimensional irrotational flow problems and provides an approximate solution to the flow problem by following simple rules, as initially set out by Forchheimer, 1900, and later refined by Casagrande, 1937. It can also be very useful tool even for problems with complex geometries, as proven in this experimental research paper.

The objectives of this experimental research paper are:

- To determine the position and shape of the flow line representing the uppermost free water surface inside the body of a dam by using a drainage and seepage tank,
- To conceptualise the flow lines system and to demonstrate that each flow line starts perpendicular to the upstream slope of the dam and that that slope is a boundary equipotential line,
- To construct an experimental flow net and subsequently to verify and analyse it by the FEA method,
- To calculate the rate of seepage through the dam body, and
- To summarise the calculations and experimental findings in a concise and readable format.

In order to achieve these objectives, an experimental flow net system inside the body of a homogeneous earth embankment dam model was formulated by using a drainage and seepage tank.

From the constructed flow net in the present experimental research paper, an attempt has been made to analyze, determine and present the following parameters:

- ✓ The pressure drop from one side of the embankment to the other,
- ✓ The seepage flow rate in each flow “channel”,
- ✓ The total seepage flow rate, and
- ✓ The pore pressure ratio, r_u , for the embankment.

KEYWORDS: Earth Embankment Dams; Flow Net; Flow Lines; Equipotential Lines; Seepage Flow Rate; Pore Pressure Ratio; Experiment Model; Pressure Drop; Experimental Conceptualisation; Permeability; Bilinear Shape Functions; Seepage analysis; Finite Element; Steady-State; Unconfined flow; Free surface; Saturated Soil; Unsaturated Soil; Hydraulic Structures; Phreatic Surface; Drainage and Seepage Tank.

INTRODUCTION

Dams are constructed to impound water for irrigation, water supply, energy generation, flood control, recreation as well as pollution control. Moreover, disastrous effects of water are significant on them.

It has been implicated that seepage plays a major role on dam failures due to its potential to cause endogenous rather than exogenous eruption of soil mass (Cedergen, 1989), as well as slope instability and failure (Sachpazis, 2013); hence embankment dams and especially earthfill dams require seepage control (Fell, 1992; Fredlund et al, 1994). Many researches indicated that failure of embankment dams due to seepage alone stands for about 25% of the total failure cases, apart from overtopping, piping, internal erosion, etc (Singh, 1995).

Free-surface, i.e. unconfined seepage problems are commonly encountered in geotechnical engineering. In these problems, the determination of the free surface and the calculation of seepage usually requires sophisticated numerical techniques, unfamiliar to most engineers.

Different methods have been identified to study the extent of seepage in earth dams. Due to its relative simplicity, flow net is the most commonly used amongst these methods.

For simple embankment dams such as a homogeneous earthfill dam with simple configurations, the configuration of a flow net is relatively straightforward in the determination of seepage quantity. However, especially for zoned earthfill dams or embankment dams with different coefficients of permeability for each zone, the complexity of seepage behaviour increases dramatically. Therefore, seepage modelling using a drainage and seepage tank as well as a finite element analysis technique can help to solve the problem promptly, thus saving funds and time, but immolating a marginal reduction of accuracy. Several authors such as Papagianakis, 1984, Lam et al, 1988, Potts et al, 1999, Rushton et al, 1979, Vandammea et al, 2013, had performed seepage analysis using either finite element or drainage and seepage tank apparatus method.

Historically, Stelzer et al, 1987, presented an introductory scheme for plotting contours that are traced along paths of constant function values. Desai et al, 1988, presented a detailed theoretical development of Residual Flow Procedure (R.F.P.) for three dimensional seepage, and a scheme for locating of the three dimensional free surface. Fan et al, 1992, presented a simple and unique method for generating flow nets based on nodal potentials and bilinear shape functions. The method reduces the work of performing a second FEM to compute the stream potentials at the nodes.

Mathematically, the process of making out a flownet consists of contouring the two harmonic or analytic functions of potential and flow line function. These functions both satisfy the Laplace equation and the contour lines represent lines of constant head, i.e. equipotentials, and lines tangent to flowpaths, i.e. streamlines. Together, the potential function and the stream function form the complex potential, where the potential is the real part, and the stream function is the imaginary part.

In this experimental study, a model of the body of a homogeneous earth embankment dam was formed by means of a drainage and seepage tank and by using Leighton Buzzard Silica sand. In this set up a flow net system was developed and the flow and seepage characteristics were studied, as outlined below.

THE EXPERIMENT MODEL SET UP

An experimental earth dam model of trapezoidal cross-section was formed from Leighton Buzzard Silica sand in the Drainage and Seepage Tank.

The general Drainage and Seepage Tank setup configuration showing the Steady State Seepage conditions through an earth dam model is presented in Figure 1. The particular experiment set up of the earth dam model is presented in Figure 2. The Flowlines through this earth dam model are also shown in this figure.

The base of the dam model was exactly 153.2 mm above the bottom of the tank. The width of the base of the dam model was exactly: 1250 mm. The upstream slope was steeper than the downstream one and its toe was approximately at the overflow outlet. The illustration of the exact dimensions and upstream and downstream slope inclinations of the experiment set up of the earth dam model is shown in Figure 3.

The dam model cross section was constructed with an upstream slope inclination of exactly 1 : 1.70 (Height : Length) and a downstream slope inclination of exactly 1 : 3.37 (Height : Length). The height of the dam model was exactly 240 mm, and the crest width was exactly 80 mm. The upstream water level was stabilised exactly 30 mm below the dam crest, and the downstream level was stabilised exactly 17.5 mm above the bottom. The height of the upstream water level was exactly 210 mm above the bottom of the dam model, and hence the water head level difference between the upstream and the downstream water level was exactly 192.5 mm (see fig. 3).

When the dam model was constructed, water was first poured into the downstream pool and only after it was full the upstream pool was filled up. The rate of filling was slow enough in order not to cause any dam model collapse due to increased internal seepage forces. Two drainage pipes, one upstream and the other downstream, were introduced to keep both water levels controlled in specific head heights as shown in figure 1, and by this way a Steady State Seepage condition through the earth dam model experiment was achieved.

When the upstream water level was stabilised, potassium permanganate grains were placed just below the surface of the upstream slope close to the glass side of the tank. The first grain was placed at the junction with the water surface and flowlines were forming over a period of appropriate and relevant to soil permeability time.

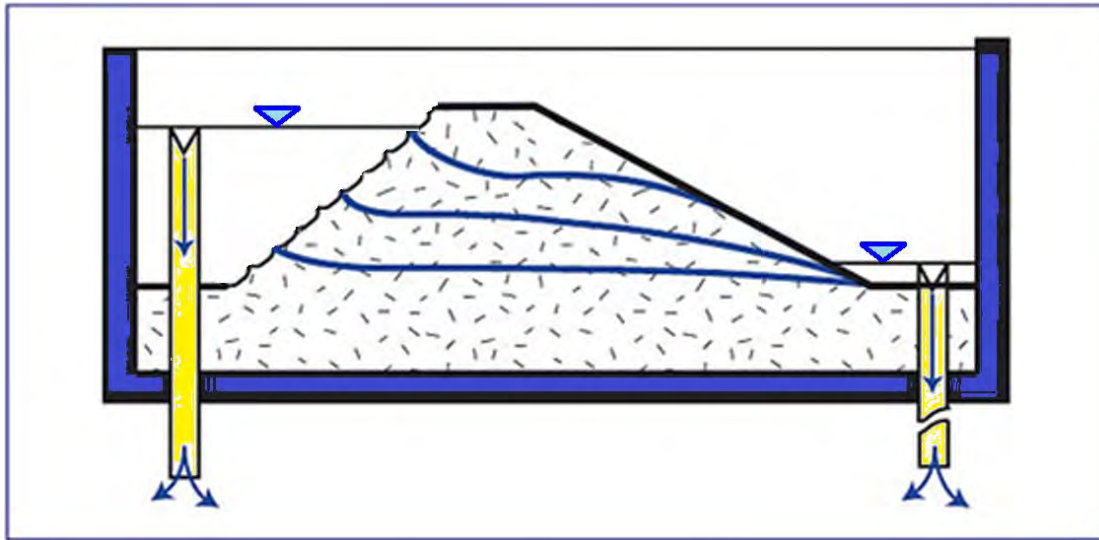


Figure 1: The Drainage and Seepage Tank setup configuration showing the Steady State Seepage conditions through the earth dam model.



Figure 2: The experiment set up of the earth dam model. Seepage flowlines through the earth dam model are shown.

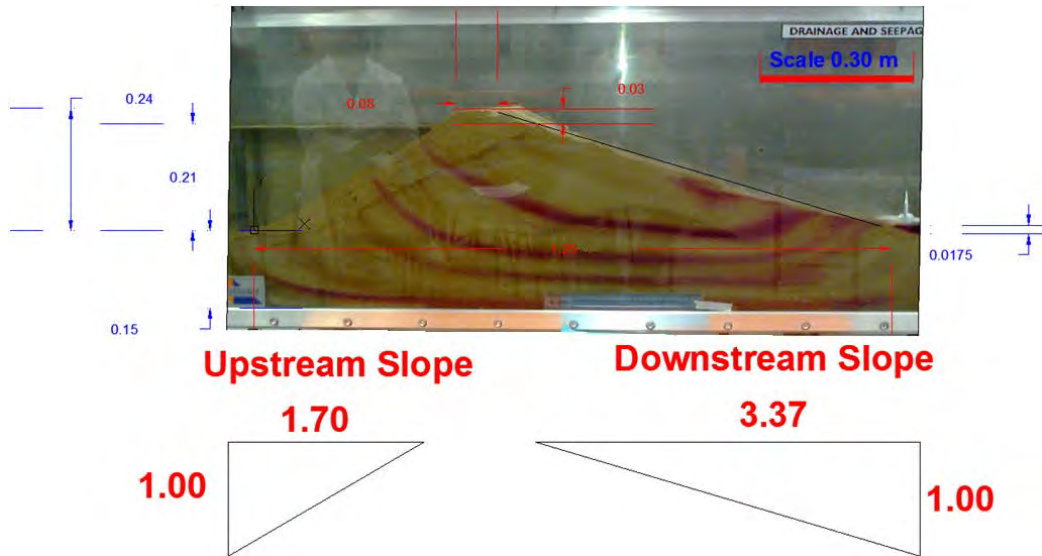


Figure 3: Illustration of the exact dimensions and upstream and downstream slope inclinations of the experiment set up of the earth dam model.

EXPERIMENTAL EARTH DAM MODEL CROSS SECTION VIEW

All important geometric data for the construction of an accurate drawing of the experimental earth dam model are summarised as follows.

Summary of geometry of the experimental earth dam model:

- Base elevation: 153.2 mm above the bottom of the tank
- Width of Base: 1250 mm
- Height (Base to Crest): 240 mm
- Crest width: 80 mm
- Upstream slope inclination: 1 to 1.70 (Height : Length)
- Downstream slope inclination: 1 to 3.37 (Height : Length)
- Upstream water level: 30 mm below crest
- Height of downstream water level: 17.5 mm above bottom
- Height of upstream water level: 210 mm above bottom
- Water head level difference (Upstream / Downstream): 192.5 mm
- Permeability of Sand in the earth dam model: Order of magnitude = 1×10^{-2} m/sec (Bowles, 1997)
- Total Area of Cross Section of the earth dam model: 0.4971 m^2 .

Using the above mentioned geometry, the experimental earth dam model Cross Section view was accurately drawn in CAD, as shown in figures 4 and 5. This accurate drawing was used for the construction of the Flow Net consisting of Flow Lines and Equipotential Lines, as presented in following sections.

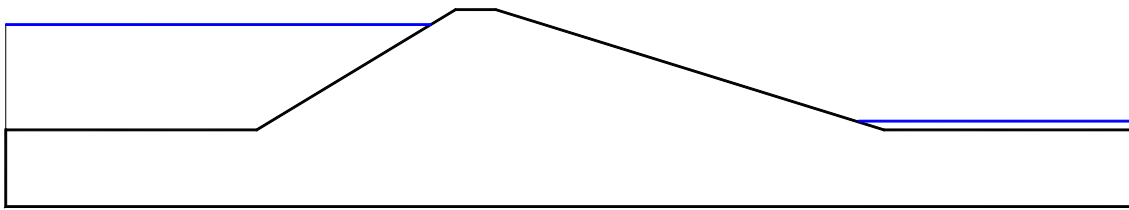


Figure 4: Cross Section view of the earth dam model drawn in CAD, based on the laboratory experiment set up.

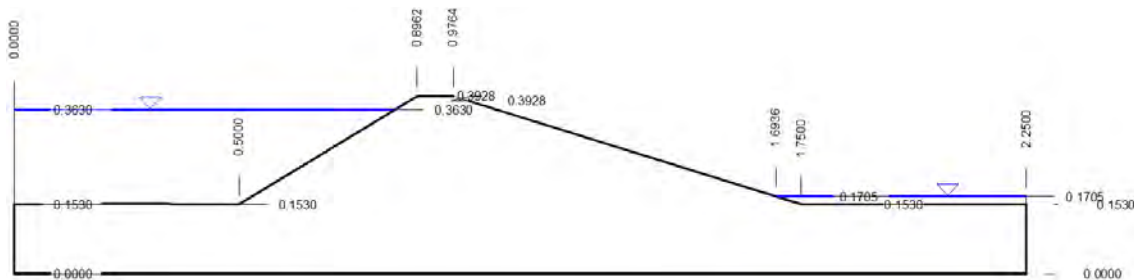
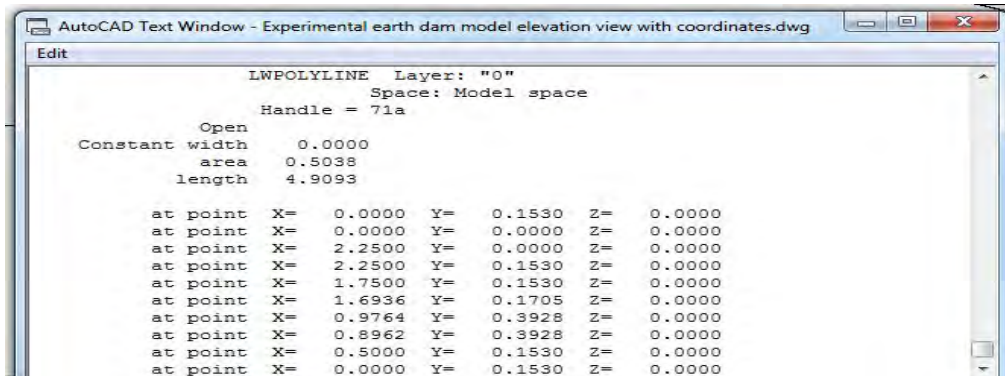


Figure 5: Cross Section view of the earth dam model drawn in CAD showing the coordinates on each node.

DETERMINATION OF POSITION AND SHAPE OF THE UPPERMOST FREE WATER SURFACE FORMING THE FLOW LINE

General

In order determine the position and the shape of the flow line representing the uppermost free water surface inside the body of the earth dam model, the procedure presented by Craig, 2004, was adopted and thoroughly followed as described below. This uppermost boundary is a free water surface and is referred to as the phreatic line or line of seepage. In order to draw the Flow Net to find the quantity of seepage through the body of the earth dam model it is essential to locate this uppermost line of seepage.

Therefore the problem of computation of the seepage loss through an earth dam primarily involves prediction of the position of the line of seepage in the cross-section, as follows.

The following are the steps in the graphical determination of the top flow line for a homogeneous dam without filters:

1. The earth dam section and upstream water level (H) to some convenient scale is drawn. Let Point-2 is the point on the upstream face coinciding with water level.
2. Let Δ be the horizontal distance between point- 2 and upstream heel of the dam. Point-1 is located at a distance of 0.3 times Δ from Point-2 on the water surface. That is distance 1 to 2, which is 0.3Δ .
3. Focus of the base parabola is located at the downstream toe of the dam, that is *Point-0* (distance *0 to 1* is d). The x - z reference axis with focus 0 as origin is selected.
4. Directrix of the parabola is at distance $2p$ from the focus 0 , where p is given by,

$$p = \frac{\sqrt{d^2 + H^2} - d}{2}$$

5. By choosing suitable values of z -ordinates (for example; $0.2H$, $0.4H$... & H) the x -ordinates of the base parabola are computed using the relationship:

$$x = \frac{z^2 - 4p^2}{2p}$$

6. All these points are joined to form the basic parabola starting from *Point-1* and concluding at a point midway between focus -0 and directrix. This parabola will be correct for the central portion of the top flow line. Necessary corrections at the entry on the upstream side and at exist on the downstream side are to be made.
7. Upstream correction: The portion of the top flow line at entry is sketched visually to meet the boundary condition there, that is phreatic line meets perpendicularly with the upstream face, which is a boundary equipotential and the phreatic line is made to meet the basic parabola tangentially at a convenient point.
8. Downstream correction (Casagrande's 1937 method): The breakout point on the downstream discharge face may be determined by measuring out L from the toe along the face. If β is the downstream slope angle then L may be computed from the following equations:

For $\beta < 30^\circ$,

$$L = \frac{d}{\cos\beta} - \sqrt{\frac{d^2}{\cos^2\beta} - \frac{H^2}{\sin^2\beta}}$$

For $30^\circ < \beta < 90^\circ$,

$$L = \sqrt{H^2 + d^2} - \sqrt{d^2 - \cos^2\beta \cdot H^2}$$

Locating the phreatic line of the earth dam model

The earth dam model cross section is shown in Figure 8.

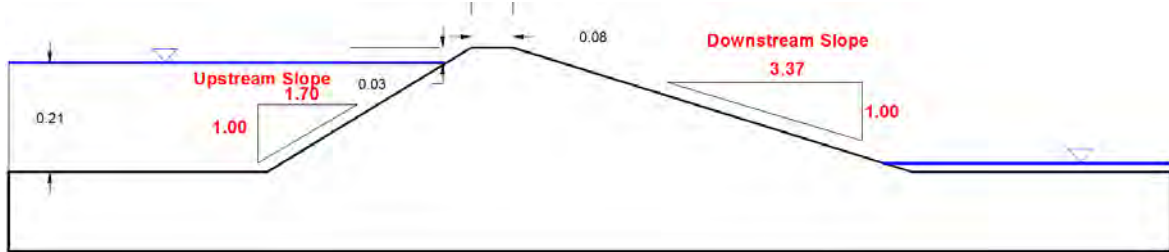


Figure 8: Cross Section view of the earth dam model for the determination of the phreatic line location.

The general diagram of phreatic line of seepage flow is shown in Figure 7. The detail procedure for plotting this phreatic line is described in previous section.

Referring to Figure 7, the following calculations are carried out using the procedure described in previous section.

- Top width of the earth dam model = 0.08 m,
- Height of the earth dam model = 0.24 m,
- Height of upstream water level (H) = 0.21 m,
- Upstream slope angle (α) = 30.466°
- Downstream slope angle (β) = 16.528°
- Base width of the earth dam model = 1.25 m

$$\Delta = 0.21 \times \frac{1}{\tan(\alpha)} = 0.21 \times 1.7 = \mathbf{0.357 \text{ m}} \rightarrow 0.3 \times \Delta = 0.3 \times 0.357 = \mathbf{0.107 \text{ m}}$$

With respect to origin, $a'(x, z) = a'(d, H)$ as shown in figure 9, (Origin is the focus of the base parabola),

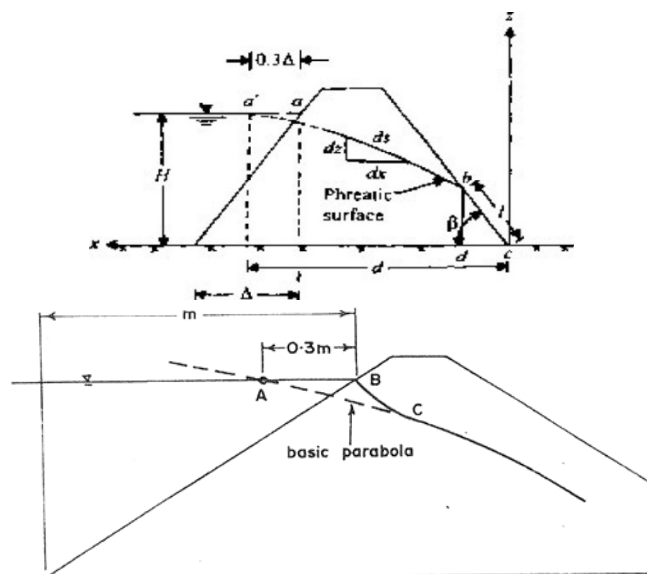


Figure 9: Basic Parabola and d, H coordinates.

$$x = d = 1.25 - \Delta + aa' = 1.25 - 0.357 + 0.107 = \underline{1.000 \text{ m}},$$

$$z = H = \underline{0.21 \text{ m}},$$

Hence, $a'(x, z) = (1.00, 0.21)$, Using Casagrande's method (For $< 30^\circ$), we have,

$$\begin{aligned} L = bc &= \frac{d}{\cos\beta} - \sqrt{\frac{d^2}{\cos^2\beta} - \frac{H^2}{\sin^2\beta}} = \frac{1.00}{\cos 16.528} - \sqrt{\frac{1.00^2}{\cos^2 16.528} - \frac{0.21^2}{\sin^2 16.528}} \\ &= \underline{0.306 \text{ m}} \end{aligned}$$

Therefore: $\underline{L = 0.306 \text{ m}}$

Finally, applying the above mentioned procedure the position and the shape of the uppermost free water surface forming the flow line referred to as the phreatic line or line of seepage was determined, as shown in Figure 10.

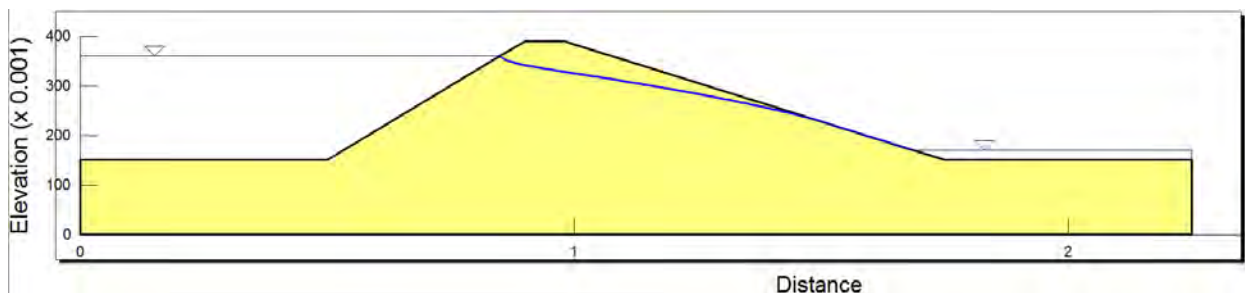


Figure 10: Position and shape of the uppermost free water surface forming the flow line, referred to as the phreatic line or line of seepage.

CONSTRUCTION OF THE FLOW NET

A *flow net* represents the graphical solution of the equations of the steady / continuous flow of groundwater. Two sets of lines constitute a flow net, which should be always orthogonal to each other. The *flow lines* indicate the direction of groundwater flow and the *equipotential lines* or *head lines* (lines which represent the constant head), indicate the distribution of potential energy. The flow nets are usually built through a trial and error procedure with sketches.

According to Casagrande (Casagrande, 1940), the following rules should be obeyed in drawing flow nets:

- Flow lines and equipotentials should always be perpendicular to each other, in a homogeneous isotropic system, and form curvilinear “squares”.
- Flow lines should always be parallel to an impermeable boundary, and Equipotential lines are always perpendicular to it.
- Flow lines should always be perpendicular to a constant head boundary, and Equipotential lines are always parallel to it.

Furthermore, for drawing flow nets the suggested procedure is as follows:

- The boundary conditions should be firstly identified, e.g. boundaries which are impermeable and which are constant head.
- Then, regions where the water is entering the system and regions where it can it should be

identified.

- Any symmetry in the boundary conditions should always be looked for.
- The number of flow channels to be used should be decided.
- By trial and error sketching, a first trial flow line must be drawn and then other flow lines must be drawn in to define all flow channels.
- Where flow channels squeeze, more closely spaced equipotential lines (higher head gradients) are required to transfer the same quantity of water through the flow channel.
- Finally, the curvilinear “squares” should be fitted together by drawing in the equipotential lines. In this process, the positions of certain flow lines may be revised, making a trial and error procedure.

Taking into account the above mentioned rules, in order to construct the correct and representative Flow Net of the experiment earth dam model, consisting of Flow Lines and Equipotential Lines, based on the laboratory experiment set up as shown on figures 2 and 3 in previous section, the following procedure was executed:

- ✓ A photograph of the experiment set up of the earth dam model was taken as shown in Figure 2, where the seepage flow lines through the earth dam model and the boundary conditions are also shown. These seepage flow lines were used as a rough guide for the flow net construction.
- ✓ From this photograph a two-dimensional scale drawing was created in CAD of the system under consideration (the laboratory experiment set up), as shown in Figure 5 and 11.

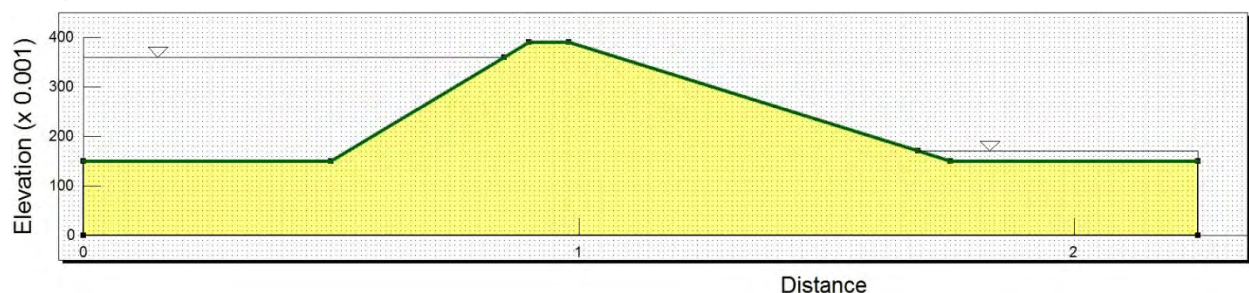


Figure 11: Detailed Cross Section view of the earth dam model.

- ✓ The boundary conditions were specified (Figure 12), i.e., the position of the upstream and downstream water table, the impermeable boundaries, the points of known head or known pressure, as follows:
 - Any constant head surface (e.g., base of a flat-bottomed reservoir) is considered an equipotential by definition, and flow lines should meet it perpendicularly.
 - The flow at impermeable boundaries should be parallel to it, i.e., impermeable boundaries are always flow lines, since flow cannot pass through impermeable boundaries, and equipotentials must meet them perpendicularly.
 - By definition, the water table is the surface where pressure, $P = 0$. Therefore, it can be an equipotential line only if it is horizontal. No matter if the water table is flat or inclining, at any point on the water table $h = z$; z is the above the datum elevation of the water table.

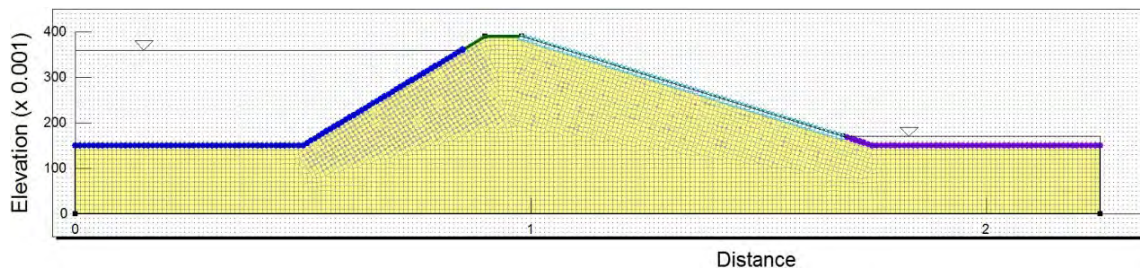
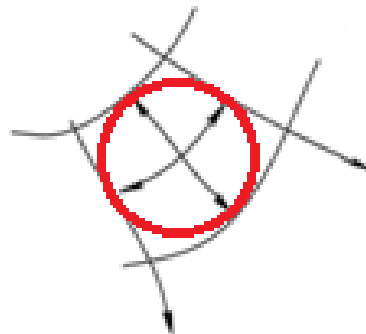


Figure 12: The boundary conditions of the earth dam model.

In case that there is no seepage flowing down to the water table, the water table can be considered as a flow line. However, in the general case where there is an inclining water table, or seepage across it, the water table is neither a flow line nor an equipotential line, and flow lines should intersect it perpendicularly.

- ✓ Once the boundary conditions have been defined, the trial sketching of flow lines and equipotentials lines was started, following the rules mentioned in the above step, making sure that the flow lines and the equipotentials always intersect perpendicularly to each other.

Trying so, it was taken into account that the flow net consist of curvilinear “squares”, with the middle line lengths of the “square” being approximately equal, as shown in the diagram below. This is particularly important if the flow net is to be used for the total seepage flow rate calculations.



By applying this procedure and rules, some trial sketching and refining by trial and error was carried out until a good set of “curvilinear squares” was achieved which satisfied the boundary conditions. These trial sketches, refined by trial and error until the accepted one was obtained, are shown in the next figures 13 to 16.

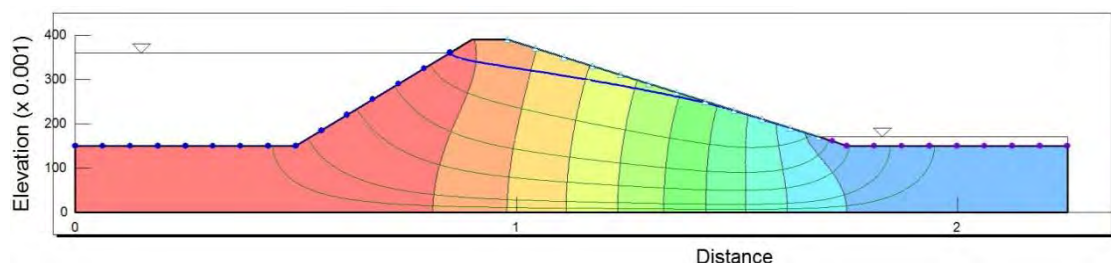


Figure 13: Trial sketching of Flow Lines and Equipotentials Lines that constitute the Flow Net consisting of curvilinear “squares”, of the earth dam model. This was NOT Accepted.

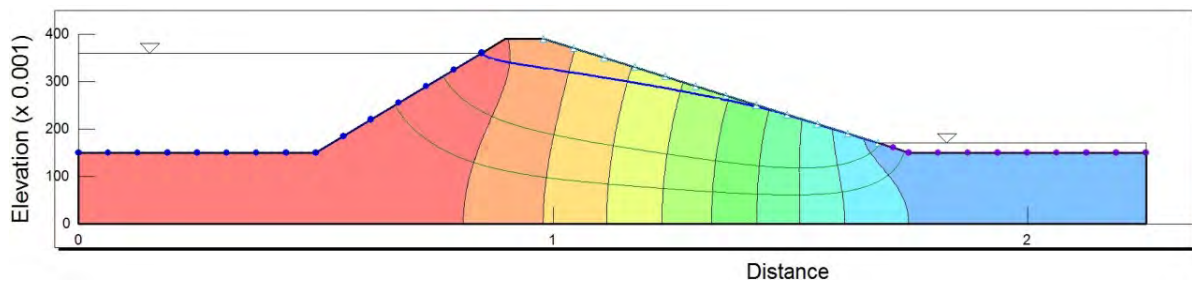


Figure 14: Trial sketching of Flow Lines and Equipotentials Lines that constitute the Flow Net consisting of curvilinear “squares”, of the earth dam model. This was NOT Accepted too.

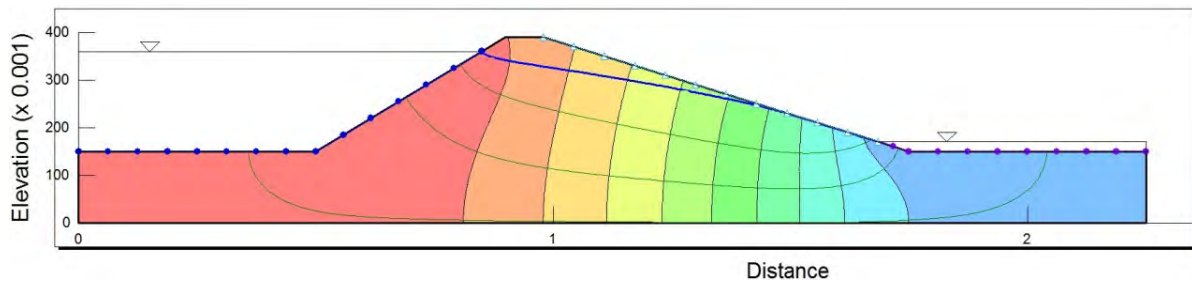


Figure 15: Trial sketching of Flow Lines and Equipotentials Lines that constitute the Flow Net consisting of curvilinear “squares”, of the earth dam model. This was ACCEPTED.

because:

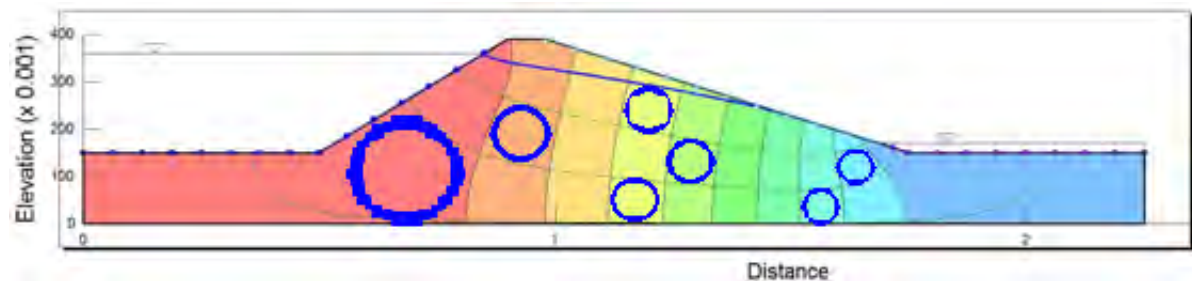


Figure 16: Circles fit almost perfectly (because there is no perfect in soil mechanics...) into the formed curvilinear “squares” of the Flow Net, of the earth dam model. Hence this Flow Net was ACCEPTED.

RATE OF SEEPAGE THROUGH THE EARTH DAM MODEL

General

Laplace’s equation governs the flow of an incompressible fluid, through an incompressible homogeneous soil medium. Combining the well known equations, i.e. continuity equation for steady state and Darcy’s equations and for the case of isotropic soil, the permeability coefficient is independent of direction (Craig, 2004).

Thus, $k_x = k_y = k$, and an equation is produced, known as the Laplace’s equation:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0$$

The two dimensional flow of water through the soil of the earth dam model is governed by Laplace's equation. Laplace's equation describes the energy loss associated with flow through a medium and is used to solve flow problems, like seepage.

However, there are certain assumptions associated with Laplace's equation, as stated below (Murthy, 2003). The limitation implied by each of these assumptions is stated within the brackets.

- Darcy's law is valid. (Flow is laminar)
- The soil is completely saturated. (Degree of saturation is 100%)
- The soil is homogeneous. (Coefficient of permeability is constant everywhere in the soil medium)
- The soil is isotropic. (Coefficient of permeability is same in all directions)
- During flow, the volume of soil & water remains constant. (No expansion or contraction occurs)
- The soil and water are incompressible. (No volume change occurs).

Laplace equation is a partial differential equation. When Laplace equation is solved graphically the equation gives flow net consisting two sets of curves intersecting at right angles known as flow lines (or stream lines) and equipotential lines, as presented in previous section.

The solution of Laplace equation requires knowledge of complex boundary conditions. Geotechnical problems usually have complex boundary conditions for which it is difficult to obtain a closed form solution. For this reason, approximate methods such as graphical methods and numerical methods are often employed. Flow net technique is a graphical method, which satisfies Laplace equation. A flow net is a graphical representation of a flow field (Solution of Laplace equation) and comprises a family of flow lines and equipotential lines, as presented in previous section.

Procedure for calculation of quantity of seepage through the earth dam model

As mentioned earlier the main application of flow net is that it is employed in estimating quantity of seepage. If H is the net hydraulic head of flow (i.e. the difference in total head between the first and last equipotential), the quantity of seepage due to flow may be estimated by drawing the flow net, which is shown in Figure 15. With reference to Figures 15 & 17, the following terms may be defined in order to estimate the quantity of seepage through the earth dam model.

N_d = Number of equipotential drops, that is, number of squares between two adjacent streamlines (Flow Lines) from the upstream equipotential to downstream equipotential

N_f = Number of flow channels, that is, number of squares between two adjacent equipotential lines from one boundary streamline to the other boundary streamline

Δ_q = Flow through one flow channel (between two adjacent streamlines)

Δ_h = Head loss between two adjacent equipotential lines

We consider a flow grid of dimension $a \times b$ (Figure 17),

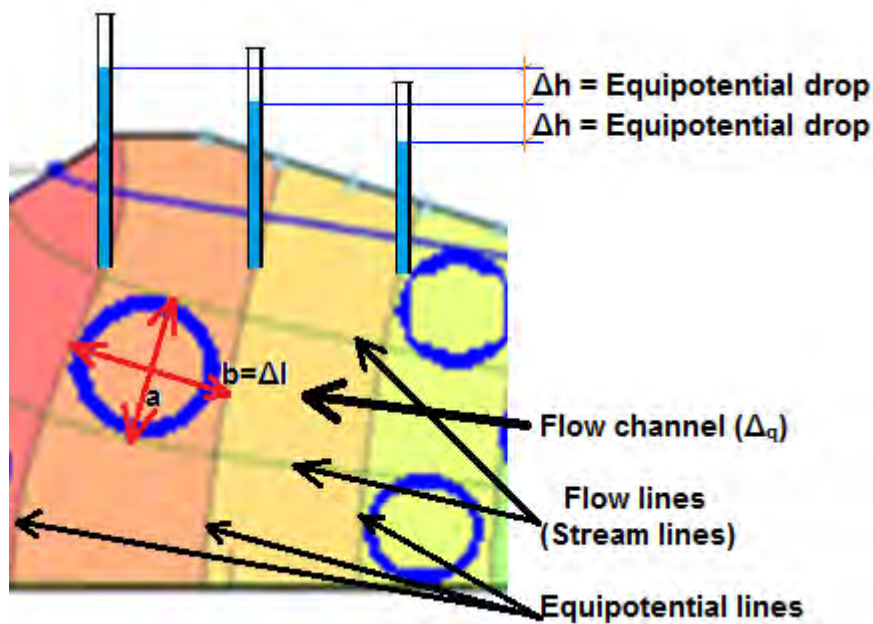


Figure 17: Flow net and its characteristics.

where,

$b = \Delta l$, which is the distance between two adjacent equipotential lines, and
 $a = A$, which is the area across the flow channel

Head loss for every potential drop: $\Delta h = \frac{H}{N_d}$

and

Hydraulic gradient: $i = \frac{\Delta h}{\Delta l} = \frac{\Delta h}{b}$

But, $\Delta h = \frac{H}{N_d}$, thus $i = \frac{H}{b N_d}$

Flow per channel (Darcy's law):

$$\Delta q = v i = k i A = k \frac{H}{b N_d} a$$

Total flow per unit length of cross section across each flow channel:

$$q = \Delta q N_f (1) = k H \left(\frac{N_f}{N_d} \right) \left(\frac{a}{b} \right)$$

If $a = b$, then,

$$\text{Total flow per unit length of cross section} = q = k H \left(\frac{N_f}{N_d} \right)$$

($\mathbf{a} = \mathbf{b}$, implies that each flow grid is square)

Thus the quantity of seepage across total length of cross section, L , of soil medium beneath the earth dam model is given by:

$$Q = q \times L = \left(k H \left(\frac{N_f}{N_d} \right) \right) \times L$$

where,

N_d = Number of equipotential drops,

N_f = Number of flow channels,

k = Coefficient of permeability, and

H = Net hydraulic head (i.e. the difference in total head between the first and last equipotential)

L = total length of soil medium beneath the earth dam model

Results of quantity of seepage through the earth dam model

For the case of the earth dam model, and with reference to Figures 15 & 17, the following data are applied:

Number of equipotential drops, N_d	=	10
Number of flow channels, N_f	=	3.01
Coefficient of permeability, k	=	1×10^{-2} m/sec
Net hydraulic head, H	=	0.1925 m
Total length of the earth dam model cross section, L	=	0.60 m (information from supplier)

Therefore:

Total flow per unit length of cross section =

$$q = k H \left(\frac{N_f}{N_d} \right)$$

Total flow per unit length of cross section:

$$q = 1 \times 10^{-2} \times 0.1925 \times \left(\frac{3.01}{10} \right)$$

Hence,

$$\mathbf{q = 0.000579425 \text{ m}^3/\text{sec per unit length}}$$

of cross section across the earth dam model.

Thus the total quantity of seepage or seepage flow rate across the total length of cross section, L , of the earth dam model is given by:

$$Q = q \times L$$

$$Q = 0.000579425 \times 0.60$$

Hence,

$$\textbf{Total } Q = \textbf{0.000347655 m}^3\text{/sec}$$

$$\textbf{for a cross section length of 0.60 m}$$

Finally, the following additional parameters are determined and presented as follows:

- ✓ The pressure drop from one side of the embankment to the other = 0.1925 m
- ✓ The seepage flow rate in each flow “channel” per unit length = 0.0001925 m³/sec
- ✓ The total seepage flow rate per unit length of cross section = 0.000579425 m³/sec
- ✓ The pore pressure ratio, r_u , for the embankment, is calculated and presented in the following section of this experimental research paper.

CALCULATION OF THE PORE PRESSURE RATIO (r_u)

Calculation of the pore pressure ratio for an embankment is highly important, as this value is extremely useful in embankment stability analysis problems (Smith, 2006).

It is highly important because typically slopes are analysed in terms of an average r_u , which can simply be calculated by averaging the r_u value within a number of “slices” and then averaging the r_u value in accordance with the area of each slice.

As such, this experimental research also assesses the validity of the ratio value obtained.

Pore pressure ratio, r_u , is the ratio of pore water pressure at any given point of the earth dam model to the weight of the soil material acting on unit area at that point (Tsuyoshi, 2006). The concept of r_u is relevant to both granular and cohesive soils.

The pore pressure ratio is expressed as:

$$r_u = \frac{u}{\gamma_w z}$$

where:

- u = Pore water pressure at any given point of the earth dam model, ($u = h_w \times \gamma_w$)
- γ_w = Unit weight of water (9.81 KN/m³)
- γ = unit weight of soil. For the Leighton Buzzard Silica sand, γ , is taken as 18 KN/m³
- z = depth of point at which pore pressure is being considered.

The following diagram in figure 18 shows a static ground water level within the earth dam model as derived from the Flow Net shown in Figure 15. In this diagram, the pore water pressure contours along with the flow lines are also presented.

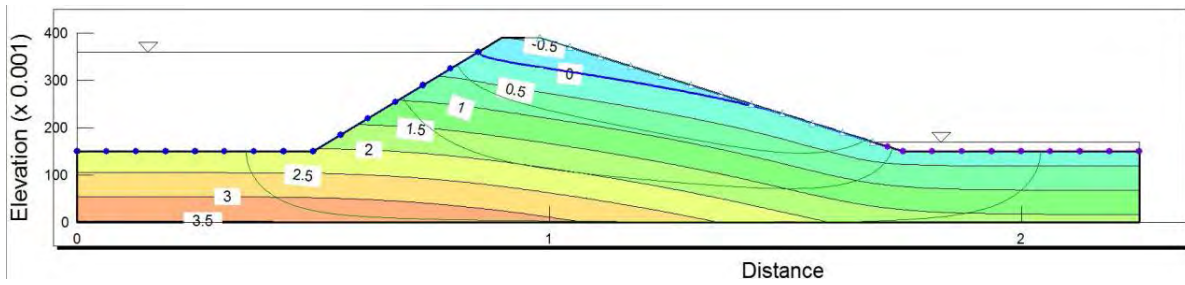
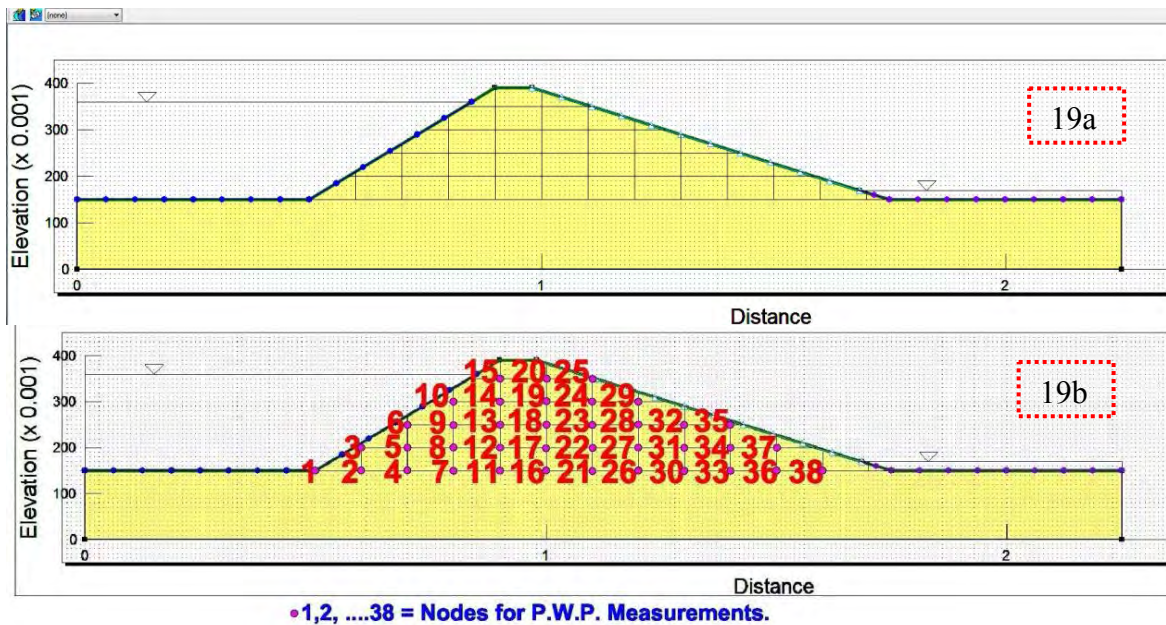


Figure 18: Position and shape of the static ground water level (uppermost free water surface) with the pore water pressure contours and the flow lines.

In order that the pore water pressure and the weight of the soil material (overburden pressure, $\gamma \times z$) acting at certain points (nodes) of the earth dam model can be calculated, a grid of such points (nodes) was created and numbered accordingly, as shown in figures 19a & 19b, and Table 1.



Figures 19a&b: Grid of points (a) and their corresponding numbers (b) in the body of the earth dam model for the pore water pressure and the overburden pressure, $\gamma \times z$, determination. (See Table 1).

Additionally, the following diagram in figure 20 shows the overlay between the grid of points (nodes) and the pore water pressure contours within the body of the earth dam model, in order to facilitate the determination of both the pore water pressure and the overburden pressure, $\gamma \times z$.

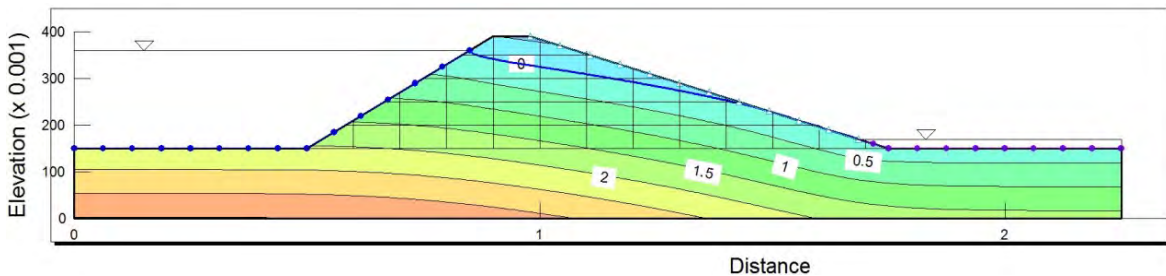


Figure 20: Position and shape of the static ground water level (uppermost free water surface) with the pore water pressure contours and the flow lines.

Table 1, shows the aggregated results of the following data in each node of interest within the body of the earth dam model:

- Node Number
- Coordinate X of Node in m
- Coordinate Y of Node in m
- Elevation Head at Node in m
- Total Head at Node in m
- Pressure Head = Total Head - Elevation Head at Node in m
- Pore-Water Pressure, u , at Node in kPa
- Depth of Node from dam surface at which pore pressure is being considered, z , in m
- Overburden Pressure, $\gamma \times z$, at Node in kPa
- Pore pressure ratio, at Node, r_u .

Table 1: Aggregated results of the geotechnical and hydraulic data in each node of interest within the body of the earth dam model.

Node No.	X (m)	Y (m) = Elevation Head (m)	Total Head (m)	Pressure Head = Total Head - Elevation Head (m)	Pore-Water Pressure u (kPa)	Depth of Node at which pore pressure is being considered, z (m)	Overburden Pressure, $\gamma \times z$ (kPa)	Pore pressure ratio, at each Node r_u
2	0.60	0.15	0.36	0.21	2.035	0.060	1.080	1.884
4	0.70	0.15	0.35	0.20	1.988	0.120	2.160	0.920
5	0.70	0.20	0.35	0.15	1.519	0.070	1.260	1.206
7	0.80	0.15	0.34	0.19	1.906	0.180	3.240	0.588
8	0.80	0.20	0.35	0.15	1.439	0.130	2.340	0.615
9	0.80	0.25	0.35	0.10	0.984	0.080	1.440	0.683
10	0.81	0.30	0.35	0.05	0.538	0.030	0.540	0.997
11	0.90	0.15	0.33	0.18	1.793	0.240	4.320	0.415
12	0.90	0.20	0.33	0.13	1.315	0.190	3.420	0.385
13	0.90	0.25	0.34	0.09	0.862	0.140	2.520	0.342
14	0.90	0.30	0.34	0.04	0.393	0.090	1.620	0.242
15	0.90	0.35	0.34	-0.01	0.000	0.040	0.720	0.000
16	1.00	0.15	0.32	0.17	1.660	0.233	4.194	0.396
17	1.01	0.20	0.32	0.12	1.176	0.183	3.294	0.357
18	1.00	0.25	0.32	0.07	0.712	0.133	2.394	0.297
19	1.00	0.30	0.32	0.02	0.232	0.083	1.494	0.155
20	1.00	0.35	0.33	-0.02	0.000	0.033	0.594	0.000
21	1.10	0.15	0.30	0.15	1.510	0.203	3.654	0.413
22	1.10	0.20	0.31	0.11	1.034	0.153	2.754	0.376
23	1.10	0.25	0.31	0.06	0.554	0.103	1.854	0.299
24	1.10	0.30	0.31	0.01	0.091	0.053	0.954	0.095
25	1.10	0.35	0.31	-0.04	0.000	0.003	0.054	0.000
26	1.20	0.15	0.29	0.14	1.349	0.171	3.078	0.438
27	1.20	0.20	0.29	0.09	0.864	0.121	2.178	0.397
28	1.20	0.25	0.29	0.04	0.403	0.071	1.278	0.315
29	1.20	0.30	0.29	-0.01	0.000	0.021	0.378	0.000
30	1.30	0.15	0.27	0.12	1.161	0.141	2.538	0.457
31	1.30	0.20	0.27	0.07	0.698	0.091	1.638	0.426

32	1.30	0.25	0.27	0.02	0.220	0.041	0.738	0.298
33	1.40	0.15	0.25	0.10	0.970	0.159	2.862	0.339
34	1.40	0.20	0.25	0.05	0.490	0.109	1.962	0.250
35	1.40	0.25	0.25	0.00	0.037	0.059	1.062	0.035
36	1.50	0.15	0.22	0.07	0.730	0.096	1.728	0.423
37	1.50	0.20	0.23	0.03	0.270	0.046	0.828	0.326
38	1.60	0.15	0.20	0.05	0.481	0.015	0.270	1.781

The average pore pressure ratio, r_u , for the whole embankment of the earth dam model have been calculated in accordance to statistical procedure and algorithms / equations that were carried out. The detailed presentation of the statistical analysis steps are not included, as this is out of the scope of this experimental research paper. However, the equations used for this calculation are presented below.

$$r_{u \text{ (Average - for - each - column)}} = \frac{h_1 r_u + h_2 r_u + h_3 r_u + h_4 r_u + h_5 r_u + \dots}{h_1 + h_2 + h_3 + h_4 + h_5 + \dots}$$

And

$$r_{u \text{ (Total - average)}} = \frac{A_1 r_{u1} + A_2 r_{u2} + A_3 r_{u3} + A_4 r_{u4} + \dots}{A_1 + A_2 + A_3 + A_4 + \dots}$$

Finally, using the above mentioned procedure the average pore pressure ratio, r_u , for the whole embankment of the earth dam model was calculated equal to 0.461.

Values of r_u vary between zero for groundwater at a considerable depth below the toe of slope and about 0.7 (Smith, 2006). A value of about 0.5 for r_u indicates that ground water conditions are close to the surface. Therefore, that was proved true in our case study of the embankment of the earth dam model, where the value for r_u is equal to 0.461, which indicates that ground water conditions are close to the surface, which indeed is.

DISCUSSION ON THE USE OF FLOW NETS IN RELATION TO DAM ENGINEERING

Flow nets are a graphical way / technique for predicting the quantity of groundwater flow from a given set of boundary conditions. Flow nets are not a rigorous determination of flow, but they can give an idea of what head looks like underground.

Flow nets are comprised of a set of equipotential or constant head lines and a set of associated flow lines, along which groundwater moves. Two adjacent flow lines form a "stream channel". Flow lines are drawn within a certain set of boundary conditions, such as surface water bodies, impenetrable rocks, etc.

Flow nets were originally used for determining how much water could flow under a dam. In case that there is a substantial amount of water flow under the body of a dam, it can create a lot of pressure on the alluvium / sediments. Over time the groundwater can erode the sediments, and the dam can collapse, causing a disastrous flood. To solve this problem analytically it is difficult, but flow nets can be used to give a graphical answer (Sachpazis et al, 2005).

Briefly, the important points of the use of flow nets in relation to dam engineering can be summarised as follows:

- Flow of water through soil masses is but one of the number of forms of stream-line flow that can be studied with the flow-net. Seepage through soil can be conveniently analysed. However, the flow-net when applied to seepage through soil is based upon the assumption that Darcy's law is valid, that the soil mass is homogeneous and the voids completely filled with water and that the quantity of water contained in any small volume does not change with time. These conditions provide the basis for derivation of a Laplace differential equation for steady seepage through soil.
- The flow-net provides a means of estimating the position of the saturation line within embankments, and the distribution of seepage forces and uplift pressures throughout. Three factors have major influence upon the position of the saturation lines and the distribution of seepage forces and pressures in earth dams and levees: (a) the distribution of soil masses within the embankment, (b) the degree of stratification of the embankment and foundation, and (c) the position and character of drains, blankets, etc. In foundations of dams, recognizing the limitations of theoretical analysis methods, the flow-net can be of help in studying the relative advantages of different design features, and in establishing economical and safe designs for earth dams and levees.
- Flow nets can be used in dam engineering for:
 - Defining top flow line
 - Estimating seepage through dam
 - Estimating seepage through foundation soils
 - Estimating seepage forces (uplift forces)
 - Controlling erosion
 - Designing of cut-off walls and ground treatment programs
 - Evaluating the effect of upstream blankets
 - Estimating pore water pressure to determine up and down-stream slope stability
 - Estimating pore water pressure distribution for effective stress analysis
 - Predicting and preventing piping failures
- Finally, Flow Nets relate with and help dam engineering by:
 - Analyzing the earth dam with a core that has a more impervious material and selecting the suitable thickness
 - Analyzing the case at which the core penetrates the impervious foundation layer and investigating the suitable depth of penetration
 - Analyzing the earth dam for different conditions and locations of filters
 - Calculating the pore pressure ratio, r_u , of an embankment for slope stability analysis problems.

SUMMARY AND CONCLUSIONS

An experimental earth dam model of trapezoidal cross-section was set up and formed from uniformly graded sand in a Drainage and Seepage Tank, in order that the seepage through a dam can be analysed and studied in relation to the position and shape of the uppermost flow line (free water surface) and the flow lines system layout.

Flow nets are a graphical way / technique for predicting the quantity of groundwater flow from a given set of boundary conditions, and although they are not a rigorous determination of flow, they can give an idea of what head looks like in an embankment.

Based on the experimental results, a Flow Net, consisting of Flow lines and Equipotential lines, was constructed and the following parameters were determined:

- Pressure drop from one side of the embankment to the other = **0.1925 m**
- Seepage flow rate in each flow “channel” per unit length = **0.0001925 m³/sec**
- Total seepage flow rate per unit length of cross section = **0.000579425 m³/sec**
- Average pore pressure ratio, r_u for whole embankment model = **0.461**
- Pore pressure ratio, r_u , at certain points (nodes) of the embankment was calculated and presented in Table 1.

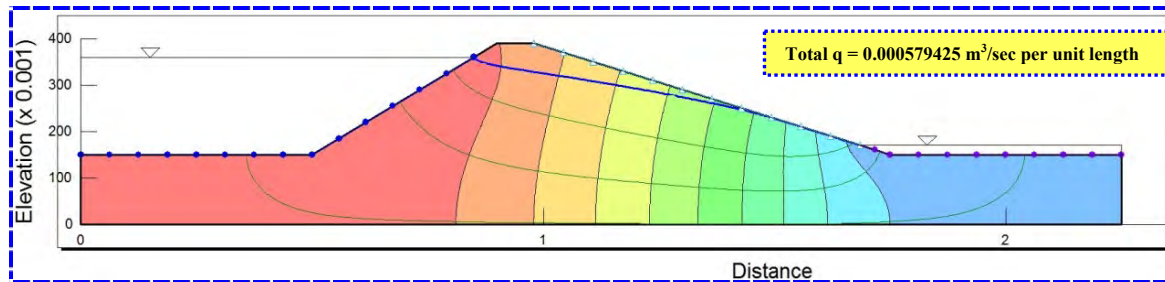


Figure 21: Accurate sketching of Flow Lines and Equipotentials Lines that constitute the Flow Net, consisting of curvilinear “squares”, of the earth dam model.

Finally, Flow Nets can help dam engineering by:

- ✓ Defining top flow line and estimating pore water pressure distribution and pore pressure ratio, r_u , for effective stress analysis and up and down-stream slope stability determination
- ✓ Estimating seepage through dam body and its foundation soils
- ✓ Estimating seepage forces (uplift forces) and controlling erosion
- ✓ Designing of cut-off walls and ground treatment programs
- ✓ Evaluating the effect of upstream blankets
- ✓ Predicting and preventing piping failures
- ✓ Analyzing the earth dam with a core that has a more impervious material and selecting the suitable thickness
- ✓ Analyzing the case at which the core penetrates the impervious foundation layer and investigating the suitable depth of penetration
- ✓ Analyzing the earth dam for different conditions and locations of filters

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